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# POVERTY: AN ORDINAL APPROACH TO MEASUREMENT

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The primary aim of this paper is to propose a new measure of poverty, which should avoid some of the shortcomings of the measures currently in use. An axiomatic approach is used to derive the measure. The conception of welfare in the axiom set is ordinal. The information requirement for the new measure is quite limited, permitting practical use.

## 1. MOTIVATION

IN THE MEASUREMENT of poverty two distinct problems must be faced, viz., (i) identifying the poor among the total population, and (ii) constructing an index of poverty using the available information on the poor. The former problem involves the choice of a criterion of poverty (e.g., the selection of a "poverty line" in terms of real income per head), and then ascertaining those who satisfy that criterion (e.g., fall below the "poverty line") and those who do not. In the literature on poverty significant contributions have been made in tackling this problem (see, for example, Rowntree [**27**], Weisbrod [**41**], Townsend [**39**], and Atkinson [**1**]), but relatively little work has been done on problem (ii) with which this paper will be concerned.

The most common procedure for handling problem (ii) seems to be simply to count the number of the poor and check the percentage of the total population belonging to this category. This ratio, which we shall call the head-count ratio $H$, is obviously a very crude index. An unchanged number of people below the "poverty line" may go with a sharp rise in the extent of the short-fall of income from the poverty line.[2]

The measure is also completely insensitive to the distribution of income among the poor. A pure transfer of income from the poorest poor to those who are better off will either keep $H$ unchanged, or make it go down—surely a perverse response. Measure $H$ thus violates both of the following axioms.

MONOTONICITY AXIOM: *Given other things, a reduction in income of a person below the poverty line must increase the poverty measure.*

TRANSFER AXIOM: *Given other things, a pure transfer of income from a person below the poverty line to anyone who is richer must increase the poverty measure.*[3]

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[2] Cf. "Its [the new Poor Law's] only effect was that whereas previously three to four million half paupers had existed, a million of total paupers now appeared, and the rest, still half paupers, merely went without relief. The poverty in the agricultural districts has increased every year" (Engels [**10**, p. 288]).

[3] Cf. Dalton's "principle of transfers" in measuring inequality; see Atkinson [**2**, pp. 247–9]. See also Dasgupta, Sen and Starrett [**9**], and Rothschild and Stiglitz [**26**].

Despite these limitations, the head-count ratio is very widely used.[4]

Another common measure is the so-called "poverty gap" (used by the United States Social Security Administration) (see [5, p. 30]) which is the aggregate short-fall of the income of all the poor taken together from the poverty line. This satisfies the monotonicity axiom but violates the transfer axiom.[5]

Though it will not be necessary to use formally the monotonicity axiom and the transfer axiom in deriving the new poverty measure (they will be satisfied anyway, implied by a more demanding axiomatic structure), the motivation of our search for a new measure can be understood by noticing the violation of these elementary conditions by the poverty measures currently in wide use.

## 2. INCOME SHORT-FALL AND POVERTY

Consider a community $S$ of $n$ people. The set of people with income no higher than $x$ is called $S(x)$. If $z$ is "the poverty line," i.e., the level of income at which poverty begins, $S(z)$ is the set of "the poor." $S(\infty)$ is, of course, the set of all, i.e., $S$. The income gap $g_i$ of any individual $i$ is the difference between the poverty line $z$ and his income $y_i$.

$$(1) \qquad g_i = z - y_i.$$

Obviously, $g_i$ is nonnegative for the poor and negative for others.

For any income configuration represented by an $n$-vector $\underline{y}$, "the aggregate gap" $Q(x)$ of the set $S(x)$ of people with income no higher than $x$ is a normalized weighted sum of the income gaps $g_i$ of everyone in $S(x)$, using nonnegative weights $v_i(z, \underline{y})$.

$$(2) \qquad Q(x) = A(z, \underline{y}) \sum_{i \in S(x)} g_i v_i(z, \underline{y}).$$

The specification of $A$ and $v_i$ will depend on a set of axioms to be proposed presently. It should, however, be noted at this stage that the form of (2) is very general indeed, and that $v_i$ has been defined as a function of the vector $\underline{y}$, and not of $y_i$ alone (along with $z$). In particular no requirement of additive separability has been imposed.

The index of poverty $P$ of a given income configuration $\underline{y}$ is defined to be the maximal value of the aggregate gap $Q(x)$ for all $x$:

$$(3) \qquad P = \max_x Q(x).$$

Since the weights $v_i$ are all nonnegative, it is obvious from (1) and (2) that:

$$(4) \qquad P = Q(z).$$

[4] The vigorous and illuminating debate on whether or not rural poverty is on the increase in India, which took place recently, was based almost exclusively on using the head-count ratio. See particularly Ojha [24], Dandekar and Rath [8], Minhas [20 and 21], Bardhan [3 and 4], Srinivasan and Vaidyanathan [37], Vaidyanathan [40], and Mukherjee, Bhattacharya and Chatterjee [22]. A remarkable amount of sophistication in correcting consumption data, calculating class-specific deflators, etc., was coupled with the use of this rather crude criterion of measuring poverty.

[5] It is also completely insensitive to the *number* of people (or the *percentage* of people) who are poor, sharing a given poverty gap.

That is, the index of poverty $P$ of a community is given by the value of the weighted aggregate gap of the poor in that community.

## 3. RELATIVE DEPRIVATION AND INTERPERSONAL COMPARABILITY

In line with the motivation of the transfer axiom, it may be reasonable to require that if person $i$ is accepted to be worse off than person $j$ in a given income configuration $\underline{y}$, then the weight $v_i$ on the income short-fall $g_i$ of the worse-off person $i$ should be greater than the weight $v_j$ on the income short-fall $g_j$. Let $W_i(\underline{y})$ and $W_j(\underline{y})$ be the welfare levels of $i$ and $j$ under configuration $\underline{y}$.

AXIOM E (Relative Equity): *For any pair $i$, $j$: if $W_i(\underline{y}) < W_j(\underline{y})$, then $v_i(z, \underline{y}) > v_j(z, \underline{y})$.*

If the individual welfare functions were cardinal, interpersonally fully comparable and identical for all persons, and furthermore if the Benthamite additive utilitarian form of social welfare were accepted, then it would be natural to relate $v_i$ in Axiom E to the marginal utility of income of person $i$. But in this paper the utilitarian approach is not taken; nor are the assumptions of cardinality and full interpersonal comparability made.[6] Individual welfare is taken to be ordinally measurable and level comparable. There is agreement on who is worse off than whom, e.g., "poor $i$ is worse off than wealthy $j$," but no agreement on the values of the welfare differences is required.

While Axiom E can be justified on grounds of a strictly concave interpersonally comparable cardinal welfare function, that is not the only possible justification. The idea that a greater value should be attached to an increase in income (or reduction of short-fall) of a poorer person than that of a relatively richer person can also spring from considerations of interpersonal equity.[7] The appeal of Axiom E is, I believe, much wider than that which can be obtained from an exclusive reliance on utilitarianism and diminishing marginal utility.

Axiom E gives expression to a very mild requirement of equity. Another axiom is now proposed, which incorporates Axiom E, but is substantially more demanding.

AXIOM R (Ordinal Rank Weights): *The weight $v_i(z, \underline{y})$ on the income gap of person i equals the rank order of i in the interpersonal welfare ordering of the poor.*

The method of constructing weights on the basis of rank orders is not new, and since the classic discussion of the procedure by Borda [6] in 1781, it has been

[6] Alternative frameworks for interpersonal comparability were explored in Sen [**29** and **31**] in which the possibility of *partial* comparability of cardinal individual welfare functions was also explored. I use this present occasion to record that in that paper Theorem 4, while valid, can be strengthened by replacing $a^*$ by $a^{*2}$ to read: "With convexity, scale independence, and strong symmetry, the aggregation quasiordering will be complete if the degree of partial comparability is greater than or equal to $a^{*2}$, where $a^* = \sup_{x,y \in X} a(x, y)$." A corresponding change should be made in the same theorem in Sen [**30**, p. 115], viz., Theorem 7*5, and in the numerical example on p. 102 there. In the present paper, however, we stick to ordinal level comparability only.

[7] On various aspects of equity considerations in welfare economics, see Graaff [**16**], Runciman [**28**], Kolm [**19**], Sen [**30**], and Pattanaik [**25**].

extensively analyzed and axiomatized in voting theory (see, especially [**11**; **12**, Ch. 13; **14**; and **17**]). Axiom R is taken as an axiom here, though it can be easily made a theorem derived from more primitive axioms (see Sen [**33** and **34**]).

There are essentially two ways of doing this. The first is to follow Borda in equidistanced cardinalization of an ordering. If $A$, $B$, and $C$ are ranked in that order in terms of their weights, and if there is no intermediate alternative between $A$ and $B$, and none between $B$ and $C$, "I say that the degree of superiority that this elector has given to $A$ over $B$ should be considered the same as the degree of superiority that he has accorded to $B$ over $C$" (Borda [7, p. 659], translated by Black [**6**, p. 157]). We know from Axiom E that if $i$ is worse off than $j$, then the weight on $i$'s income gap should be greater than on $j$'s income gap. Using Borda's procedure combined with appropriate normalization of the origin and the unit, we arrive at Axiom R.

The second is to take a "relativist" view of poverty, viewing deprivation as an essentially relative concept (see Runciman [**28**]). The lower a person is in the welfare scale, the greater his sense of poverty, and his welfare rank among others may be taken to indicate the weight to be placed on his income gap.[8] Axiom R can be derived from this approach as well.

We turn now to the relation between income and welfare, since Axioms E and R are in terms of welfare rankings, whereas the observed data are on income rankings. There are, of course, good reasons to think that sometimes a richer person may have lower welfare than a poorer person, e.g., if he is a cripple, and this may raise interesting issues of equity (see [**32**, Ch. 1]). When dealing with a general measure of poverty for the community as a whole, however, it is not easy to bring such detailed considerations into the exercise. Axiom M proceeds on the cruder assumptions that a richer person is also better off. Furthermore, the individual welfare relation is taken to be a strict complete ordering to avoid some problems that arise with rank-order methods in the case of indifference. This last assumption is less arbitrary than it may at first seem.[9]

AXIOM M (Monotonic Welfare): *The relation* $>$ *(greater than) defined on the set of individual welfare numbers* $\{W_i(\underline{y})\}$ *for any income configuration* $\underline{y}$ *is a strict complete ordering, and the relation* $>$ *defined on the corresponding set of individual incomes* $\{y_i\}$ *is a sub-relation of the former, i.e., for any* $i, j$: *if* $y_i > y_j$, *then* $W_i(\underline{y}) > W_j(\underline{y})$.

## 4. CRUDE INDICATORS AND NORMALIZATION

In Section 1, references were made to two measures of poverty currently in use. The "head-count ratio" is the ratio of the number of people with income $y_i \leqslant z$, to the total population size $n$:

[8] This can be axiomatized either in terms of the welfare rank of the person among the poor (as in Aiom R) or in terms of that among the entire population (see Axiom R* in Section 6 below). Both lead to essentially the same result if correspondingly normalized (see Axiom N).

[9] The poverty index $P$ to emerge in Theorem 1 is completely insensitive to the way we rank people with the same income. See equation (15) below.

(5) $$H = \frac{q}{n}.$$

The other measure—the poverty gap—is silent on the number of people who share this gap, but can be easily normalized into a per-person percentage gap $I$, which we shall call the "income-gap ratio:"[10]

(6) $$I = \sum_{i \in S(z)} g_i/qz.$$

While the head-count ratio tells us the percentage of people below the poverty line, the income-gap ratio tells us the percentage of their mean short-fall from the poverty level. The head-count ratio is completely insensitive to the *extent* of the poverty short-fall per person, the income-gap ratio is completely insensitive to the *numbers* involved. Both should have some role in the index of poverty. But $H$ and $I$ together are not sufficiently informative either, since neither gives adequate information on the exact income distribution among the poor. Further, neither measure satisfies the transfer axiom, or the requirement of putting a greater weight on the income gap of the poorer person (axiomatized in Axiom E given Axiom M).

However, in the special case in which all the poor have exactly the *same* income level $y^* < z$, it can be argued that $H$ and $I$ together should give us adequate information on the level of poverty, since in this special case the two together can tell us all about the proportion of people who are below the poverty line and the extent of the income shortfall of each. To obtain a simple normalization, we make $P$ equal $HI$ in this case.

AXIOM N (Normalized Poverty Value): *If all the poor have the same income, then $P = HI$.*

## 5. THE POVERTY INDEX DERIVED

The axioms stated determine one poverty index uniquely. It is easier to state that index if we number the persons in a nondecreasing order of income,[11] i.e., satisfying:

(7) $$y_1 \leqslant y_2 \leqslant \cdots \leqslant y_n.$$

THEOREM 1: *For large numbers of the poor, the only poverty index satisfying Axioms R, M, and N is given by:*

(8) $$P = H[I + (1 - I)G],$$

*where G is the Gini coefficient of the income distribution of the poor.*

[10] Another measure—let us call it $I^*$—is obtained by normalizing the "poverty gap" on the total income of the community:

(6*) $$I^* = Iqz/nm^*,$$

where $m^*$ is the mean income of the entire population.

[11] If there is more than one person having the same income, (7) does not of course determine the numbering uniquely. But the formula for the poverty index specified in Theorem 1 yields the same $P$ no matter which numbering convention is chosen satisfying (7).

PROOF: By Axiom M, there is a way of numbering the individuals satisfying (7), such that:

(9) $$W_1(\underline{y}) < W_2(\underline{y}) \ldots < W_n(\underline{y}).$$

For any person $i \leqslant q$, there are exactly $(q + 1 - i)$ people among the poor with at least as high a welfare level as person $i$. Hence by Axiom R:

(10) $$v_i(z, \underline{y}) = q + 1 - i.$$

Therefore, from (2) and (4):

(11) $$P = A(z, \underline{y}) \sum_{i=1}^{q} g_i(q + 1 - i).$$

In the special case in which all the poor have the same income $y^*$ and the same income gap $g^* = z - y^*$, we must have:

(12) $$P = A(z, \underline{y})g^*q(q + 1)/2.$$

But according to Axiom $N$:

(13) $$P = \left(\frac{q}{n}\right)\left(\frac{g^*}{z}\right).$$

Therefore, from (12) and (13):

(14) $$A(z, \underline{y}) = 2/(q + 1)\, nz.$$

From (11) and (14), it follows that:

(15) $$P = \frac{2}{(q + 1)nz} \sum_{i=1}^{q} (z - y_i)(q + 1 - i).$$

The Gini coefficient $G$ of the Lorenz distribution of incomes of the poor is given by (see Gini [**15**] and Theil [**38**]):

(16) $$G = \frac{1}{2q^2m} \sum_{i=1}^{q} \sum_{j=1}^{q} |y_i - y_j|,$$

where $m$ is the mean income of the poor.

Since $|y_i - y_j| = y_i + y_j - 2 \min (y_i, y_j)$, clearly

(17) $$\begin{aligned} G &= 1 - \frac{1}{q^2m} \sum_{i=1}^{q} \sum_{j=1}^{q} \min (y_i, y_j) \\ &= 1 + \frac{1}{q} - \frac{2}{q^2m} \sum_{i=1}^{q} y_i(q + 1 - i). \end{aligned}$$

From (15) and (17), it follows that:

$$P = \frac{1}{(q + 1)nz}\left[zq(q + 1) + q^2m\left(G - \frac{q + 1}{q}\right)\right]$$

which in view of (5) and (6) reduces to:

$$(18) \qquad P = H\left[1 - (1 - I)\left(1 - G\left(\frac{q}{q+1}\right)\right)\right].$$

For large $q$, (18) yields (8). This establishes the necessity part of Theorem 1, and the sufficiency part is easily established by checking that $P$ given by (18), and for large $q$ by (8), does indeed satisfy Axioms R, M, and N.

## 6. POVERTY AND INEQUALITY

The role of the Gini coefficient of the Lorenz distribution of the incomes of the poor is worth clarifying. This is best done by posing the question: what measure of inequality would follow from the same approach as used here in deriving the poverty measure?

The poverty index was derived by making use of the more primitive concept of the aggregate gap $Q(x)$. It should be noticed that given the weighting system precipitated by Axioms R and M, the value of $Q(x)$ is the same for all $x \geqslant z$, so that $P$ defined by (3) as $\max_x Q(x)$ can be taken to be $Q(x)$ for any $x \geqslant z$ and not merely $x = z$. This is because Axiom R makes the weight on the income gap $g_i$ of person $i$ equal to the number of people *among the poor* who are at least as well off as person $i$. The inclusion of people above the poverty line $z$ does not affect the value of $Q$ since the weight on their income gap $g_i$ is zero in view of Axiom M.

This is reasonable enough in measuring poverty, but if we now shift our attention to the measurement of inequality, we would like to consider the income gaps of people above the poverty line as well. Furthermore, the income gaps should be calculated not from the exogenously given poverty line $z$, but from some internal characteristic of the income configuration $\underline{y}$, possibly the mean income. Variations in these lines will transform an absolute poverty measure into a relative measure of inequality.

To do this, we replace $z$ by the mean income $m^*$ of $\underline{y}$. Further, the weighting given by Axiom R is modified to include all the people whether poor or not.

AXIOM R*: *The weight $v_i(z, \underline{y})$ on the income gap of person i equals the number of people in S who are at least as well off as person i.*

Axiom R* will require that the weight $v_i$ on the income gap of person $i$ should be $(n + 1 - i)$.

The problem of measurement of inequality and that of poverty can be seen to be two intertwined exercises. The measure of inequality corresponding to the measure of poverty $P$ can be defined in the following way.

DEFINITION: The measure of inequality $\eta$ corresponding to the poverty index $P$ as specified in Theorem 1 is the value obtained in place of $P$ by replacing $q$ (the number of poor) by $n$ (the total number of people in the community), and replacing $z$ (the poverty level) by $m^*$ (the mean income of the community).

THEOREM 2: *The measure of inequality* $\eta$ *corresponding to the poverty index approximates the Gini coefficient for large n.*

The proof is obvious from (15) and (17) replacing $q$ by $n$, and $z$ and $m$ by $m^*$, in the formulations of $P$ and the Gini coefficient (now redefined for the whole community). This is also checked by putting $H = 1$ and $I = 0$ in $P$ as given by Theorem 1.

Thus the poverty measure $P$ obtained in Theorem 1 is essentially a translation of the Gini coefficient from the measurement of inequality to that of poverty.[12]

A diagrammatic representation of $G$ and $P$ is provided in Figure 1. Line $OGB$ is the Lorenz curve, while $OB$ is the line of equal division. The Gini coefficient $G$ is given by area $OGB$ divided by area $OAB$. The slope of the line $OD$ gives "the poverty line" in these normalized units, and $OE$ is the number of the poor. The poverty measure $P$ can be seen to correspond to area $OGF$ divided by area $OEI$. The difference between the two lies in (i) the slope of line $OD$ ("the poverty line") being different from the slope of line $OB$ (the normalized mean income), and (ii) counting only the poor, i.e., $OE$, in the poverty measure, as opposed to all, i.e., $OA$.

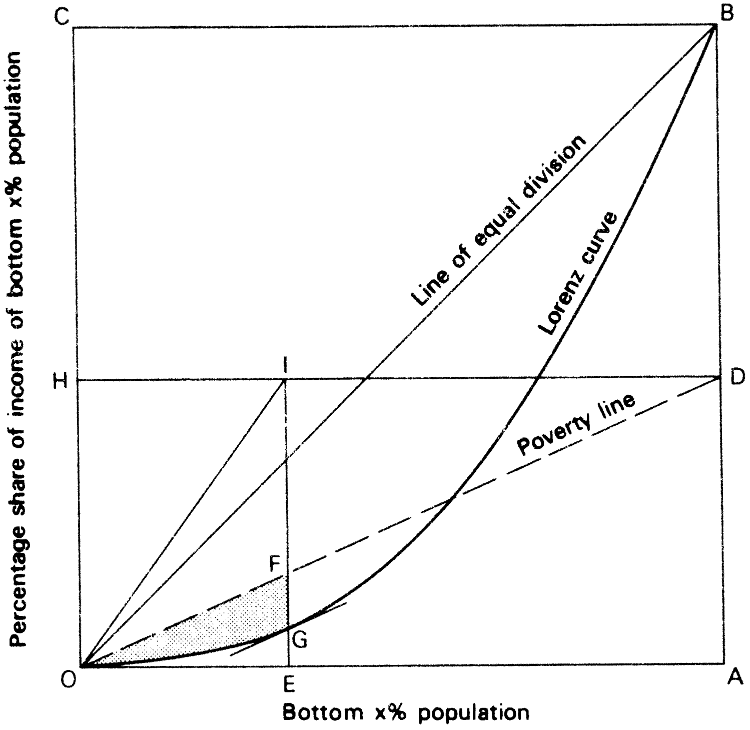


FIGURE 1.

[12] An axiomatization of the Gini coefficient as a measure of inequality can be found in Sen [**34**]. The more primitive axiom system used there leads to $R^*$ as a theorem without being taken as an axiom in itself.

The rank-order weighting form of the Gini coefficient $G$ and the poverty measure $P$ can be understood intuitively by considering the area under the curve $OGB$, which the Gini numerator leaves out, i.e., what $(1 - G)$ includes. The poorest man's income is included at every point and if there are $n$ persons his income comes in $n$ times. On the other hand, the highest income is included in the area under $OGB$ exactly once at the point $A$ when everyone is counted in, i.e., the richest man makes it exactly once more than the camel can get through the eye of a needle. The $i$th poorest man comes in at the $i$th point of observation and has his income included for the remaining $(n - i)$ observations as well, thereby having his income counted in $(n + 1 - i)$ times. This produces the rank order weighting through the mechanism of the Lorenz curve, and it is this remarkable coincidence that makes the Gini coefficient give expression to the normative value judgement of weighting according to ordinal ranks satisfying Axiom R* (and Axiom E) given Axiom M. The same way of intuitively understanding the result regarding the poverty measure can be easily suggested by considering the number of times the gap between the slope of $OD$ (the poverty line) and the slope of $OGB$ (the income of the poor) gets counted in.

## 7. INTERPRETATION AND VARIATIONS

The poverty index proposed here turns out to have quite an easy interpretation. The measure is made up of the head-count ratio $H$ multiplied by the income-gap ratio $I$ augmented by the Gini coefficient $G$ of the distribution of income among the poor weighted by $(1 - I)$, i.e., weighted by the ratio of the mean income of the poor to the poverty-line income level. One way of understanding its rationale is the following: $I$ represents poverty as measured by the proportionate gap between the mean income of the poor and the poverty line income. It ignores distribution *among* the poor, and $G$ provides this information. In addition to the poverty gap of the mean income of the poor reflected in $I$, there is the "gap" arising from the unequal distribution of the mean income, which is reflected by the Gini coefficient $G$ of that distribution multiplied by the mean income ratio. The income-gap measure thus augmented to take note of inequality among the poor, i.e., $I + (1 - I)G$, is normalized per poor person, and does not take note of the number of people below the poverty line, which could be minute or large. Multiplying $[I + (1 - I)G]$ by the head-count ratio $H$ now produces the composite measure $P$.

While this is perhaps the easiest way of interpreting the poverty index $P$, it must be borne in mind that its justification lies in the axioms used to derive it. The multiplicative form chosen in Axiom N, though simple, is arbitrary. Axiom M, perhaps justifiable in the absence of detailed information on the poor, is objectionable when much is known about individual members of the group, e.g., that cripple Mr. $A$ while richer than robust Mr. $B$ is less well off in some sense (see [**32**, pp. 17–20]). Finally, Axiom R follows Borda's procedure of cardinalizing an ordering by treating rank numbers as weights. This is, of course, also arbitrary, though frequently used in other contexts as the popularity of several variants of the rank order procedures of voting indicates. The justification can be either in terms of intensity of preference being surmised from rankings only by using a version of "insufficient reason" (following Borda), or in terms of an essentially relativist conception of poverty. Axiom R may not be acceptable to many since there is arbitrariness in making the weight on person $i$'s income gap equal his poverty rank. Even with a *given* level of income, a person's poverty weight will go down if a richer poor becomes poorer than him. The advantages and defects of the rank-order system are clear enough.

A few properties of $P$ may be worth pointing out. It lies in the closed interval $[0, 1]$, with $P = 0$ if everyone has an income greater than $z$, and $P = 1$ if everyone has zero income. In practice, of course, $P$ will never equal unity, both because there are subsistence requirements (so that for each $i: y_i > 0$) as well as because even in very poor economies the class system ensures the prosperity of some (so that for some $i: y_i > z$).

Note also that when all the poor have the same income, i.e., $G = 0$, the lower the income of the poor, the closer will $P$ approach the head-count measure $H$, and the larger the proportion of the poor, the closer will $P$ approach the income-gap measure $I$.

Some variations of the normalization procedures may be worth considering. First, if the weights on income gaps are all reduced by one-half, i.e., the income gap $g_i$ of the $i$th poorest is taken to be $(q - i + \frac{1}{2})$, then (8) holds not only for large $q$ but for any $q$. However, for measuring poverty of any sizeable community, the two procedures do not make any real difference.

Second, even retaining the weighting procedure, the normalization reflected in Axiom N can be changed. In particular, the poverty measure can be made to depend also on the *ratio* of the mean income of the poor to the mean income of the entire community (Sen [**33**, equations (8) and (9)]). This would give the poverty measure wider coverage. For example, exactly the same number and income distribution of the poor will have a *higher* poverty index if the income of some people above the poverty line falls even without taking them below the poverty line.[13] In contrast, the measure $P$ is completely invariant with respect to changes in the income of people *above* the poverty line and depends only on the incomes of the poor. This does not, of course, prevent us from defining the poverty line $z$ taking note of the entire distribution of income (e.g., a higher $z$ for the United States than for India), but once the poverty line has been specified, the poverty measure $P$ depends only on the incomes of the poor.

## 8. CONCLUDING REMARKS

(i) The measure of poverty $P$ presented here uses an ordinal approach to welfare comparisons. The need for placing a greater weight on the income of a poorer person is derived from equity considerations (Axiom E) without necessarily using interpersonally comparable *cardinal* utility functions. Ordinal level comparability is used to obtain rank order weighting systems (Axiom R) given a monotonic relation between income and welfare (Axiom M).

[13] Consider the "mean-dependent measure" $P^* = Pz/m^*$.

(ii) The poverty measure $P$ obtained axiomatically in Theorem 1 corresponds to the Gini measure of inequality in the sense that replacing the poor by the entire population and replacing the poverty threshold of income by the mean income would transform $P$ into $G$. This poverty measure $P$ contrasts sharply with the crude measures of poverty used in the statistical literature on the subject and in policy discussions. Unlike $H$ (the percentage of people below the poverty line), $P$ is not insensitive to the extent of the short-fall of income of the poor from the poverty line. Unlike $I$ (the percentage average short-fall of the income of the poor from the poverty line), $P$ is not insensitive to the number below the poverty line.[14] And unlike any conceivable function $\Psi(H, I)$ of these crude measures, $P$ *is* sensitive to the exact pattern of distribution of the incomes of the poor.

(iii) Throughout this paper income has been taken to be a homogenous magnitude represented by a real number. The framework developed here can be extended to multicommodity cases as well, evaluating the consumption of commodity $j$ by person $i$ in terms both of the price of $j$ and the income rank of $i$, basing the calculation on Fisher's [**13** and **18**] "commodity matrices."[15]

(iv) If one accepts the ordinal welfare interpretation of the rationale of the Gini coefficient (Axiom R*), then one might wonder about the significance of the debate on the non-existence of any "additive utility function which ranks income distributions in the same order as the Gini coefficient" (see Newbery [**23**, p. 264], Sheshinski [**36**], Dasgupta, Sen, and Starrett [**9**], and Rothschild and Stiglitz [**26**]). Evidently, $-G$ is not an additive function of individual incomes, nor is it strictly concave or strictly quasi-concave (as is obvious from equation (17)). Axiom E and specifically Axiom R* precipitate the equality-preferring result noted in [**9** and **26**]; but the ordinal weighting of the Gini coefficient cannot be cast into the strictly concave utilitarian framework, or into any other social welfare function that makes marginal weights sensitive to the exact values of income (as opposed to their ordinal ranks). The same applies to the poverty measure $P$ proposed here.

(v) Finally, it should be pointed out that any system of measurement that takes note only of *ordinal* welfare information must be recognized to be deficient by an observer who is convinced that he has access to cardinal interpersonally comparable welfare functions. If such cardinal information did obtain, the fact that $P$ should throw away a part of it and use only the ordering information must be judged to be wasteful. On the other hand, it is much more difficult to agree on interpersonally comparable cardinal welfare functions than to find agreement on welfare rankings only. The approach proposed here, while deficient in the sense described, also demands less. It is a compromise in much the same way as the Borda method of voting is, in making do with rankings only and in slipping in an

[14] The alternative definition of the income-gap measure $I^*$ is sensitive to the number below the poverty line, but it is also sensitive to the incomes of people *above* the poverty line. Furthermore, $I^*$ is insensitive to the *distribution* of income among the poor.

[15] The use of such an approach has been explored, as an illustration of a general system of real income comparisons with explicit treatment of distribution, in Sen [**35**].

assumption of equidistance to get numerical weights. The data requirement in estimating the poverty measure $P$ is, as a consequence, quite limited.

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